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THE MATHEMATICS TEACHER

Volume XXIX Number 3

Edited by William David Reeve

Crises in Economics, Education, and Mathematics*

By E. R. HEDRICK

University of California at Los Angeles

1. INTRODUCTION. That there exists a crisis in our economic life is perhaps more evident than that there exist crises in educational affairs and in the teaching of mathematics in particular. What I desire most tonight is not to bewail these crises, but to discuss their causes and the part that we ourselves may play in such a vital stage in the history of this country of ours. What underlying causes exist? What may be done? What is our own duty?

It is easy to dismiss these questions, and to say that the individual is helpless either as a voter or as a single unit in that great public which creates what we call public opinion. In large measure that is indeed true. Yet the public mind does exert effective influence; just such meetings as this are the focal points from which much influence may be exerted. Nor can one of you disclaim duty in regard to each of the apparently widely separated fields —economics, education, mathematics— which I have mentioned in the subject of this talk. Even if there be those among you who are so engrossed in—say—mathematics that he has no time nor energy left for consideration of the others, the fact remains that he can not escape the interconnections that exist today between these three great fields of human interest. Indeed I shall try to point out that the crises in economic affairs and in education are themselves connected, and that each of these directly contributes to what I shall call the crisis in mathematical teaching in our schools and colleges.

2. *The crisis in economics.* Of economics and of political systems I shall talk very little apart from effects upon the others. Before I have finished, you will see that I have no panacea for world-ills, and that I am not what is called a "reformer." That evils exist is all too obvious. But though I am a professor, I am slow to talk or even think of change of our social system: I am not ready to "fly to evils that we know not of." In order to show the connection of the present economic crisis with these other crises, I shall state a few of the things that seem to me to be obvious.

We all realize that rapacity and greed have gone too far, that the general welfare has been neglected or forgotten, and that a minority of men in high places—not politicians alone, but also some bankers, some lawyers and jurists, some business men, and some educators—have betrayed the trust which the public has bestowed upon them, and have brought into disrepute (undeservedly) the very high callings which they have disgraced. The re-

* An address delivered before the National Council of Teachers of Mathematics at Denver, Colorado, on June 29, 1935.

sulting lack of confidence is surely one great cause of present economic ills.

Does this belie my assurance that I have no wish to urge upon you social reform or governmental change? In a sense, I have said it only to establish a sort of background. The facts are perfectly well known. I have added nothing to what is on public record. My real point is that just such statements of fact are frequently interpreted as radical utterances, and the speaker is called socialist or communist if such facts are mentioned. To do so, that is, to marshall what facts there are, to examine them dispassionately, and to seek what meagre conclusions seem possible, is perfectly normal and not vicious. Yet there is a spirit amounting almost to prudery concerning these things. They who call themselves conservatives most loudly—misusing that word to disguise what is only selfishness—these pseudo-conservatives see *red* wherever there is only *light*. A real conservative is not bothered by the light, by knowledge of the facts, but only those are bothered who have something to conceal.

An immediate effect of this upon education (for I intend to talk mainly of education) is apparent on every college campus, in every school system, and in much of the less thoughtful public utterance and public printing. What I regard as only a symbol of this widespread effect is the apparent belief that all college professors, or at least most of them, are communists, radicals, revolutionists.

I mention this only as a symbol. Among the other effects of the same sort are the taboos that exist everywhere in our schools. I shall not stop to defend college professors against the wild charges that are hurled against them. Aside from your graciousness to me, I assume that the salvation of college professors appears to most of you to be either unimportant or maybe impossible. It goes much further than that. This outcry is in essence an outcry against all *light*, against knowledge of the facts, against enquiry; in short, against *education*. To exclaim against mistaken conclusions is just; to exclaim against intemperate speech is always sound; but to exclaim against dissemination of knowledge and against careful thought regarding the conclusions to be drawn is directly to attack education.

It seems to me that much that is said by pseudo-conservatives is then not conservatism at all, but only fear of exposure, fear of public enlightenment, fear of all light. If that be conservatism, then I would not qualify, for I am committed, as are most men in educational life, to investigation, enlightenment, and stimulation of public thinking. On the other hand, if conservatism means deep aversion to sudden change, if conservatism means to suffer the ills we know rather than to fly to evils that we know not of, if conservatism means careful investigation of everything that is novel, then I have strong claim to be called "conservative." Such conservatives, the true conservatives, are not opposed to education and to public enlightenment. Yet the forces of pseudo-conservatism, the purely selfish desire to prevent criticism of what is and has been, are opposed to true education. They threaten today its very foundation: free investigation, free thought, free speech, a free-press.

These forces proceed not from persons or groups who are in any serious sense conservative, but precisely from those persons or groups who have acquired special privileges or vested rights, and who seek, quite aside from any reason or any philosophy, be it radical or conservative, to hold those privileges and to maintain those rights. Throughout the whole course of human history, entrenched privilege and vested rights have always led to abuses of power by the more unscrupulous, and to a vehement insistence upon the maintenance of those rights based not on a philosophy of conservatism, but only on greed and selfishness.

3. *The educational crisis.* In such ways, then, the present economic crisis, and the

threat that exists against entrenched privilege, affects education, and creates a crisis, for it strikes at free investigation and free speech, which lie at the very basis of true education.

There is, of course, a direct effect of the depression on education that results from lack of public revenue and from objection to taxation for any purpose. This, however, is not so dangerous to education as a whole, in my opinion, as is the more insidious attack based upon actual hostility to true education, hostility to enlightenment, which arises from fear of consequences of knowledge and of thought.

Shall we then meet the opposition to such enlightenment by ourselves urging radicalism and the introduction of courses of inflammatory character? There appears to be precisely such a tendency. This I would regard as just as dangerous to true education as is that other tendency.

It is no secret to you that there is a group in the educational world that is pressing strongly for the complete reorganization of school-systems, and that seeks in particular to make the curriculum of secondary schools, at least, center primarily on the social sciences. One aim of this group that has been expressed publicly is to guide the oncoming generation toward a "proper" form of social organization. It is evident that the leaders of this group have in mind a particular type of social organization which they wish to have instilled into the minds of our children.

To accomplish this, they who form this group are willing and eager to center the whole attention of the public secondary schools on these "directed" social studies, and to eliminate entirely the sciences, the modern languages, and mathematics from their curricula, at least in so far as these subjects are to be required for any large body of students.

Thus our whole secondary-school program is faced with two utterly opposed, but still similar, tendencies and forces. The one, which calls itself conservative, is really the reactionary and selfish group that clings to special privilege; they desire to prevent any criticism of our present economic system or even of the abuses of it, in all schools and colleges. They often represent themselves to be particularly "patriotic," which is in this connection only another disguise for self-interest.

To me, these are no more the foes of education than are those who seek to use our public class-room to instill in childish minds what they have decided upon as an ideal social system. Either of these programs is a perversion of true education. Each group seeks to indoctrinate the coming generation with beliefs held by their leaders, and to prevent a free investigation of the real facts, or independent thinking about other systems.

Propaganda has no place in public education, as I understand the word propaganda. I may define propaganda to be the effort to instill ideas without regard to their truth. To do so presupposes some ulterior motive, something to be gained by those who manage the machinery. Public education must not sink to a level where its main aim is to sacrifice the child for the gain of those who . . . *have something to gain.*

4. *Education for life.* There was a slogan of education that is now half-forgotten in this welter of opposing propaganda. It is that education properly speaking is *preparation for life*, or, indeed, *life itself.* For such a program I have had hearty sympathy.

It is under this general motive, I presume, that some would class the effort toward a nearly exclusive emphasis on the social studies. If education is for life, and if the social organization is intimately knit with living, it may seem that concentration upon the social studies would conduce toward wholesome education.

Citizenship and right thinking on public questions are among the goals that seem justified, particularly in public, tax-supported, schools.

Such goals are good if they be real, and

if they are not over-emphasized to the exclusion of other goals. They cease to be healthy stimuli to proper growth and become cancerous abnormalities if they yield to the influence of either type of propaganda which I have mentioned.

Social studies, properly understood, and freed from the constant menace of propaganda, are well-fitted for the true purposes of education and for life, both for the State and for the individual. However, we should not make the error of mistaking the mere label "social" for the reality. There is really little that is "social" in some studies that are so labelled. There is much that is truly social in studies that are not so labelled.

Thus mathematics contains much that is distinctly social in its setting and in its implications. I have contrasted elsewhere, at some length, the social implications of such things as compound interest on money (which is mathematical) with such as the wars of the Roses. The one is not so labelled but is actually social in all its bearings. The other is labelled "social," but its effects and implications are as lacking in social qualities as is the worst monstrosity in formalized factoring. Life insurance is a mathematical study, but its far-reaching social effects are legion. By contrast, the study of Roman Law is scarcely "social" at all.

5. *Mathematics and mathematical teaching.* I have talked long without apparently reaching the subject nearest to your immediate work and interest: mathematics and mathematical teaching. This is not through lack of interest in it, but because I wished to bring out how the crises in economics and in education, that threaten all that we have known both of social organization and of public secondary education, have produced also the present crisis in the teaching of mathematics.

I shall not need to convince you either of the merits of mathematics, nor of the crisis that threatens its extinction as a required subject in major curricula of secondary schools. Too often gatherings of this kind degenerate into the glorification to one's own brethren of the virtues of their own particular subject. I have indeed already made the one point that I would most emphasize to you and to the world: that mathematics has many social effects and implications, that it, quite as much as many another field, contributes directly toward an understanding of the life about us, and of the civilization in which we live.

To dwell upon this theme would be to carry coals to Newcastle, before this audience. You know these elements in mathematics. I have myself stated them in considerable detail in public utterances and in printed papers. I have also warned you, teachers of mathematics, years ago, that we would face just such a crisis in mathematical teaching as that which is upon us unless we make clear to our students and to the world these real values for life and for social affairs that are to be found in mathematics.

It is more important that I insist to you upon our faults rather than upon the virtues of the subject itself. A while ago, you agreed by nod or gesture when I said that entrenched privilege and vested rights have always led to abuse of power. Have you perhaps thought of that with respect to our own situation? Or do you wish to apply it *only to others than ourselves?*

Mathematics has held a favored place in the curricula of secondary schools, and in the requirements for entrance to colleges. The public, even, has given us its trust, and has believed that mathematics is one of the fundamental subjects in all education. We have had, as few departments have had, preferred position, entrenched privilege, vested rights, in so far as secondary education is concerned.

Have we then escaped the universal human weakness of those who enjoy special privilege? Have we never abused the power represented by requirement of our courses in curricula? Do we, when

threatened, fall back only upon the claim of conservatism, as do those others, in order to maintain for ourselves those vested rights?

Power—be it in the economic world or in mathematical prestige in the schools—should bring with it responsibility. Public trust should lead to sincere and thoughtful effort to serve to the utmost. Have you, have we, done that? Have we striven to bring out the elements in mathematics that make for the public welfare? Do you yourselves doubt the existence of elements that serve the public welfare?

Will you now, threatened with the loss of vested rights, at last determine to serve social ends rather than our own petty convenience? Thus, it is much easier (particularly if your training consisted mainly in football) to give formal work in algebra than it is to show its connections with life. Have you, will you, teach how quantities may be related to other quantities, in the actual world, in preference to many pages of formalized problems in factoring? Will you prefer to say that factoring is a sacred part of the established algebraic routine which must be maintained to its fullest extent because it would be scandalous to upset tradition?

Have you maintained the traditional "theorems" (so-called) about ratio and proportion instead of showing the children the ways in which ratios work, as in elementary trigonometry, and in problems of surveyors, draughtsmen, carpenters, and mechanics, not to speak of housewives? Do you prefer the stock methods of solving simultaneous quadratic equations to the graphical relationships that represent ellipses and hyperbolas? Have you made clear to students that the most important case of geometric progression is that of compound interest on money, or that the most intriguing case of geometric progression is the disappearance of radioactive substances? Have you insisted more upon the so-called "incommensurable" cases in geometry than you have upon the immediate importance of the congruence theorems in the building of bridges and of steel-framed buildings? Do your students know more about the properties of fractional exponents than they do about the value of logarithms in computation; and do they know that these subjects are essentially identical?

I have mentioned quickly a few of the specific instances in which *our* power may have been abused. In some cases we have been led to do something because we ourselves take delight in the intricate processes of reasoning involved: this is to serve our petty vanity and our own pleasure rather than the child's interests or the public welfare. In some instances we have done things because they were simply easier to do, as when we dwell upon the formal exercises in factoring. In general, we have earned the reputation, which mathematics itself does not deserve, of teaching a formalized and essentially meaningless discipline. May I not indeed compare this with the bad reputation which our social system has gained, though it deserves a better one, through the neglect of the public welfare by some who enjoyed special privilege?

Such has been our failure. It has followed rather closely the ordinary course of abuse of power by those who possess vested rights. We have ourselves to blame, and it will be our own fault if the present threats against mathematics as a school subject are carried out to a total elimination of mathematics as a required part of any school curricula.

No selfish appeal based upon our supposed rights will avail us greatly. What will affect the situation, if anything, will be for you and teachers everywhere, in mathematics, in science of every sort, in modern language, to take an active part in the present crises in education as a whole, and to see to it that the revision of curricula is sound from the standpoint of those principles which I have emphasized.

Education must serve the public welfare. It must serve the public as an organized State, and it must serve the individ-

ual for his own life and in his living. At all costs, it must be kept free from propaganda. In mathematics, we must make clear the fundamental effect upon the whole of our civilization. Its effect on life must be made known to students, to the public, to sincere leaders in education. Its social values must be emphasized rather than ignored.

If you, the teachers of mathematics, do not do these things, if the formal symbolistic exercises continue to shut out the real mathematics for which they are only a preparation, then mathematics in the schools is doomed, doomed by your own acts, doomed as are doomed in every field those who base claims for power and privilege solely on vested rights rather than on service for the public good.

If that happens, then, maybe, after a period of storm and stress, of badly organized and perverted education, we may come back in a saner generation to mathematics for the public welfare, to a mathematics that does not disgrace the name. Just as I hope for our body politic, so here in the field of mathematical education, I hope most fervently that sane counsel will prevail before it is too late, and that we shall not have to pass through the fire in order to be purified.

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A High School Teacher's View on the Training of Teachers of Secondary Mathematics*

By MARTHA HILDEBRANDT
Proviso Township High School, Maywood, Illinois

ONE HEARS so much about changing the curriculum; about introducing into our high schools new courses, in some of which the subject matter is embarrassingly vague; about tests and measurements and laboratories to help the pupil adjust himself. . . . I hesitate, unable to choose a word for that to which the pupil must adjust himself and not at all certain just what the pupil has to adjust. One reads about differentiated courses and incidentally also about integrating courses. Is it not possible that each new fashion in education is just another attempt to improve the teaching in the schools of the respective states from which you and I come.

Before continuing, I wish to state that this is merely a point of view, for I have sent out no questionnaires, nor have I canvassed in any manner any large body of teachers on this subject. I have had no experience in preparing young people to teach high school work. I have assisted in selecting teachers for my department. From my own experience in high school teaching and in working with high school teachers, I have drawn conclusions concerning the training of teachers of mathematics which I wish to discuss with you.

I suppose a high school teacher's preparation begins before he starts to school, but it would be similar to the preparation for so many pursuits that we can safely pass along to the high school course. Perhaps some of us are in accordance with many of the colleges which accept for admission any course which enables a pupil to graduate from high school, but I much prefer to see the high school lay as good a foundation as possible in English, foreign language, history and science as well as mathematics for our prospective teacher of mathematics. I would ask that during the college years this acquaintance be extended to include as much as can be arranged of English writing and literature, foreign language and literature, more history, economics, philosophy, music, fine arts and other general subjects. Such a general background is not only valuable in a general way in making contacts with pupils and parents, but in adapting our teaching to the experience and background of the various individuals in our classes.

May I discuss one of two of these courses in a little more detail? For instance, the courses in English. After all, just how valuable is the instruction of a teacher of English if the pupil spends the next period with a teacher of mathematics who violates many or few of the rules of correct English? Is it not inconsistent to insist on accuracy in your own subject and ignore accuracy in connection with those things with which every person of education should be familiar? Or let us consider it from another point of view. Several years ago I attended an educational meeting at which a member of a department of education in one of our leading universities recommended eliminating mathematics from the high school program. He had the same antipathy for languages. I am not certain, but I gained the general impression that he felt that such mathematics as was absolutely necessary could be taught in the courses in English and social science for which he showed more enthusiasm. I would suggest such a thorough background in English for our teachers of mathematics that if it is ever necessary to cast courses

* Read before the General Meeting of the National Council of Teachers of Mathematics at Denver (Colo.) on June 29, 1935.

out of the curriculum, our administrators will spontaneously agree that it is to the best interest of the child to have the necessary English taught in the courses in mathematics. After all, an accurate knowledge of mathematics is created by accurate explanations and clear statements, on the part of the pupil as well as the teacher, and this decidedly requires the use of good English.

Or let us give a moment to the cause of language. I believe that subject matter is really valuable, not in isolated form but in relation to other school subjects and the daily lives of the pupils. Call it "integration" if you like. If members of classes of mathematics are studying German or French, why not use these languages in reading mathematics and make them seem really useful. In our school, the language courses are stressing the ability to read fluently and so from time to time, as our finances permit, I am adding foreign language books to the mathematics shelf of our library. These usually pertain to recreational mathematics, but I am hoping presently to include textbooks in mathematics used by various countries in their pre-university training. It is interesting to note that the books which we have, are being used.

A mathematics teacher, to be really valuable, must have more than a bowing acquaintance with his own subject. In some states fifteen semester hours of mathematics are required of teachers of mathematics and these may include not to exceed six hours done in high school beyond plane geometry. Colleges vary in the number of semester hours allotted to freshman mathematics but we can presume that this minimum may include trigonometry, college algebra, analytic geometry and possibly something of the differential and integral calculus. You will agree with me, that at best the required preparation is woefully meager and decidedly elementary. To really appreciate the value and the power of these courses, our prospective teacher should have had work in which he used as tools, the subject matter to which he has been introduced. So he should be encouraged to browse in the introductory courses to the various fields of mathematics. There is certainly a wide range for selection . . . solid analytics, theory of equations, theory of probability, statistics, mathematics of finance, history of mathematics, introduction to the theory of numbers, introduction to higher algebra, modern goemetry, synthetic projective geometry, analytic projective geometry, applied calculus, definite integrals, differential equations, the courses in mechanics, limits and series and many others. These courses after all, are not very advanced and they would not mislead our candidate into considering himself as a research worker and a scholar and hence too learned to bother with high school pupils. It is my opinion that a wide knowledge of these elementary courses in the various fields of mathematics is far more profitable to the prospective high school teacher than specialization in any one field, especially if he is fortunate enough to be in the class of a teacher who has planned his work for the prospective high school teacher as well as for the prospective research man and so gives his students opportunities for additional work along the lines of their interests. It tends to give our candidate a wider insight into the many kinds of mathematics and its many uses; a more thorough and first hand comprehension of what topics taught in high school mathematics are important and of the how and why of their importance; it tends to give him power, enthusiasm and interest in his own subject matter, without which no teacher can really teach.

I would not stop there with my preparation, but would insist on work in the allied fields of physics, mechanics, astronomy, navigation, surveying and so forth. A friend of mine, well grounded in mathematics once registered for a course in statistics offered in a school of commerce. Her classmates were much better versed in

commercial subjects than she, yet she found this course much easier than they did. I have often wondered whether our friends who specialize and work in the allied fields, even economics, would not find their work much simpler if they had better ground work in mathematics. The pupils in our classes have a variety of backgrounds and interests. They will be the physicists, the economists and other scientists of tomorrow. Is it not possible that a high school teacher, with a general knowledge of these allied subjects, might not only arouse their interest and attract their attention to better work but also help them to be better prepared for the fields of work they choose and demonstrate to them how related and interdependent these departments really are?

If teachers are not born but made, our departments of education claim the privilege of making them. They try to care for all needs by offering a wide range of courses. Some of these are good, others leave much to be desired. Such courses must be chosen carefully or they will prove a poor investment both in time and money. Methods courses are of little value when they are taught by people with little or no experience in secondary school work; by people who are not familiar with the harrowing circumstances under which secondary teaching is sometimes done; or by people who must offer new methods, theories or philosophies which they claim are the salvation and cure-all of whatever evil is the greatest present worry, in order that they may receive publicity or fame of a sort as educators. There is need for professional training in education. This need is to some extent supplied by courses on the philosophy and psychology of education with special reference to high schools and methods of class room procedure. No doubt, somewhere in these courses time is devoted to teacher behavior, attitude toward the pupil, the school and the community.

The education courses of the greatest immediate value are no doubt those which treat the aims, organization and methods of teaching the subject matter of secondary school mathematics, perhaps even the subject matter itself; courses which consider why mathematics should be taught to adolescents and how it should be done; courses which study the foundations of algebra and geometry as viewed from a mathematical, psychological and perhaps even from the philosophical and historical point of view. Anyone who has taught algebra and geometry to prospective teachers can regale one by the hour with stories of how little the members of their classes know about high school subject matter. In fact, this statement need not be limited to prospective teachers, for frequently one finds teachers with years of experience who are not aware of the power of the subject matter they are teaching and lack a mastery of it which would enable them to apply it. More than one teacher teaches his first and second and sometimes more years of algebra and geometry and keeps just one jump ahead of the class in subject matter (sometimes not even that!). Is it not pitiful to see a teacher refuse to accept a correct proof or solution because it is not the one shown in the teacher's hand book and he has not the power of his subject matter to realize that there is frequently more than one correct manner of arriving at a conclusion? To be sure, there are some good texts on the market and beginning teachers as well as football coaches and prospective administrators may follow these, problem for problem and page for page, skipping such material as is beyond their comprehension and so cover the work for the year. By "cover" I mean get through it after a fashion. A good teacher should not be dependent upon the textbook, but should have so varied a background and the power to use it that he is capable of making the course turn the textbook into a living, breathing thing full of meaning. Power in and enthusiasm for subject matter is a wonderful help in teaching character, citizenship and even in teaching boys and girls.

The subject matter courses become particularly valuable when they include a detailed study of units, not only the most difficult ones but as many as possible. I would suggest that in such a course the prospective teacher begin his file of material to be used in making his class preparations from day to day. This should include methods of approach, outlines, references to allied subject matter, tests, problems, illustrations likely to arouse interest, problems fitted to remedial work, treatment for brighter students and problems which will open up new avenues of thought for them, pictures, advertisements, puzzles, sources of material and so forth, in fact, anything which might be of value in the treatment of a unit. To be sure, not all of this material could be used each time the topic is taught. Also with time, some of it would prove of little value and could be destroyed. Other material could be added. But a file of this kind, begun during the practise teaching period of a teacher's life not only becomes valuable, but does much to improve daily preparations and thus produces better teaching. I find my own files invaluable and never complete. I would not have time to begin them during the year, but I do find opportunity to add to them.

I can perhaps make my point here much clearer with an illustration. One day while visiting a school, the principal recommended to me the class of Mr. X whom he considered an exceedingly good teacher. The principal informed me that Mr. X had a master's degree from a leading university and came well recommended. The class was one in geometry and they were beginning the theorem concerning the right triangle in which the perpendicular is drawn from the right angle vertex to the hypotenuse to prove three things: that the triangles are similar to each other and to the given triangle, that the perpendicular is the mean proportional between the segments of the hypotenuse and that either side is the mean proportional between the entire hypotenuse and the segment of the hypotenuse adjacent to that side. Let us refer to our file and see what material is on hand for this topic. Here is a suggestion to approach it by ignoring the textbook entirely. The figure is constructed in any one of several ways. Then one has three triangles which can be shown to be right triangles and similar to each other. John proves them similar in one way and Susan in another. With a little praise children love to show originality and incidentally review more than one theorem. Then we set up a chart of the corresponding sides which we know are in the same ratio. Now the fun begins. How many proportions can be set up? Which are the same except for the application of one of the properties of proportions and which property is applied? There are of course the three continued proportions which, when stated in English, give us the theorem. Translating them into clear and concise English is incidentally giving the English department a helping hand. Moreover, when a pupil has chosen and discarded words in an effort to say exactly what he means, no more and no less, it is a fair guess that he understands the theorem and has made some progress toward mastering it. If the first right triangle was drawn by inscribing an angle in a semicircle, it is possible to restate the same theorem in the language of the circle. Then there is a wealth of construction material. Of course, there are immediately two ways of constructing a mean proportional and it does not take the pupils long to decide when the one method has an advantage over the other. The theorem can also be used to construct a third proportional. With a slight manipulation of the proportions in the chart it is possible to set up quite a series of formulae, such as the following: the square of the hypotenuse; the diagonal of a square; and the product of the hypotenuse and the perpendicular to it is equal to the product of the two sides. This last one can be treated again, earlier or later, in the unit on areas. With these formulae at

our disposal, it is possible to build any number of word problems, no two alike, and so review our freshman Algebra. There are other things in the file but this is enough to show us that the theorem has as many aspects as "Listen to the Mocking Bird" has variations. Summarizing briefly, we have reviewed any number of theorems, we have expressed ourselves in excellent English, we have had some construction work, we have built formulae, we have reviewed our Algebra both in solving word problems and using different algebraic processes in so doing.

Now let us return for our visit. The period began. After a preliminary statement concerning the work he proposed to do, the teacher said, "State the theorem for today." There was little or no success in achieving the statement. Greatly concerned, the teacher suggested that they spend several minutes in learning the statement. After due time was spent in rote memory work, there were a few fairly satisfactory statements as far as words were concerned. How much they meant to the staters, I hesitate to say. The teacher quickly drew a figure and in stilted fashion the triangles were shown to be similar by the method used in the text. Then the teacher professed himself dissatisfied with the work of the class and they were asked to write the theorem out in full, immediately, during class time. This writing was in most cases done with the book open and the pupil leafing back and forth to copy the reasons for the statements made by the book. While this was being done, the teacher walked up and down the aisles offering a word here and there. I presume this was a form of supervised study.

Before I left this school, I investigated the training of this teacher. He had several years of teaching experience and a master's degree in education. The courses presented for it were a motley array. I wish I could remember the names of all of them. Courses such as "Problems of the Rural Schools," "The Junior High School," courses for prospective principals, almost seventy hours worth. In mathematics he had completed the calculus, in the allied subjects there was very little. If he had had a well rounded background, thorough training in subject matter and better, more suitable professional training courses, he might have taught the lesson in the same manner, but we hope not.

I doubt whether these courses, whether they be of general nature, in mathematics, in the allied fields or in the department of education are in themselves satisfactory. They are of little value if taught by poor teachers, teachers without enthusiasm for or power in the subjects they teach. For after all teachers are imitative. It is easier for them to do as they are shown than to do as they are told. Good college teaching tends toward good high school teaching and sends into our colleges and out into the world more capable students. Sometimes I think that if more of our colleagues in the universities and colleges were more careful in planning their class room teaching, they might eventually themselves reap the benefit thereof. It is just possible that with them as with us, there are those who are not really prepared to teach the courses it is their privilege to teach and that they fail to take the matter seriously and to work at making an excellent job of it. There are those among them who upon completing a course do not have the feeling that their students have achieved definite mental growth, power, maturity—call it what you will—in the work of the semester. I do not mean that our prospective teachers should as students be babied along and their work simplified and diluted beyond all bounds. To the contrary, it should be organized and enriched in such a manner that students find themselves working to full capacity or beyond in spite of their good intentions not to. The other day I heard a man in speaking of a certain engineering school say, "whether a man has an average or good mind, he comes out of that school able

to use the things he knows." They must have good teachers in that school.

In some respects teachers are born and not made. There are certain characteristics which our candidate for a teaching position must possess and which no department can really give him. I refer to character, personality, tact, humanness, a desire to give of one's time, energy and talents unselfishly, a sympathetic attitude toward students (with discretion), the ability to explain, the ability to coordinate, and other traits too numerous to mention which are the foundations on which good teachers are built.

Perhaps you feel that I am asking a great deal in the preparation of a teacher: a well rounded background, plenty of work in mathematics and the allied subjects, and certainly more careful thorough work in professional training. A four year course does not provide time for all that. There is no reason why it should for after all a teacher is not made in four years, possibly not even in five or six. There is no law against making the best of the four years leading to the bachelor's degree and returning again and again after teaching time has begun, for more professional training and certainly more work in the elementary higher mathematics and in the allied subjects. And then you are thinking, "Does she not know that a mathemathics teacher has a great many duties in a high school beside teaching mathematics?" All of us who have taught high school work know only too well that teaching five possibly six classes a day, with periods ranging from forty minutes to an hour in length, with the attendant checking of papers and daily preparation, and with extra time given to remedial work, is only a part of the work expected of the high school teacher. There are also the other duties which frequently loom greater than the class room work to the administrator, the board and the public, at least they are duties which these people often seem to understand and appreciate more than the actual class room work. I refer to such things as assisting in the coaching of football, basketball, tennis, golf or any other sports; managing the subscriptions or advertising for the school paper or year book; managing the ushers and taking tickets for school plays, athletic games, concerts and the like; taking charge of extra-curricular activities such as class, departmental, social and general clubs and being responsible for their parties, dances and the like; policing the halls; performing the required clerical work which piles up so rapidly in our high schools these days; filling out detailed information on college entrance blanks; advising small groups of students and of course always bearing in mind the latest hobby of our educators. Thousands of teachers throughout the land are spending precious time on apparently trifling duties which could be performed just as well by people with far less training, and they are fretting because they cannot find the time or energy to do those things well which they have been trained to do well and which are fundamentally most important to the education of those entrusted in their care. Then why have a thorough education if you cannot find the opportunity to use it to its full capacity? Why be well prepared and fret when your executive officers apparently assume that you are not capable and spend much time providing you with "busy work"? Why demand that teachers be cultured people?

The trouble with this "busy work" of our administrators is that it minimizes the importance of the fundamental purposes of our teaching. While incidentally, our purpose is to develop our pupils socially, physically, psychologically, vocationally; principally our purpose is to train them in an understanding of modern culture. To be sure, we are not entirely certain as to just what is modern culture but we do know that it is changing and is somewhat different today from the culture of earlier peoples. A cultured man in ancient Greece was quite different from one in Venice at its height and he in turn

differed widely from the typical man of culture in the mid-Victorian period. One characteristic seems to be present in the culture of each—an invariant if you like—namely, an interest in and knowledge of the various forces that shaped and controlled his environment. Paul tells us that the Athenians, then near their cultural peak, were always interested in some new thing. Petronius, the leading man of culture in Rome at the time of Nero, had been a governor of one of the Roman Provinces in Asia. He was keenly aware of the importance of the Roman armies in the life of his time. In Shakespeare's picture of Venice, a favorite form of greeting on the part of the aristocrats was "What is the news along the Rialto?" They knew what was going on and were keenly interested financially and otherwise, in the voyages of trade and discovery of their time. The leaders in the court of Elizabeth were keenly interested in the development of the new world and participated in many phases of this development. In fact, some of this development was based upon the invention of certain instruments of navigation which grew out of mathematical principles, considered advanced mathematics at the time. The courts of France were interested in the development of science, even when it was getting so complicated that they could not understand all of it. Franklin's popularity at the court of France was due in a large part to his scientific achievement. There are many more examples of leading people of culture being interested in the economic and scientific developments of their times.

Modern industry is the child of science. Science and methods of science dominate our environment. No claim to culture today is valid unless a person making such a claim understands something of science which dominates our environment. Certainly one of the important changes in modern culture is the relative increase in scientific thinking. We have come to emphasize reasoning, the why of things, the scientific attitude as of primary importance as contrasted to earlier emotional basis. Where the earlier cultures emphasized the importance of acceptance of authority and rule, tradition and the sentimental appeals, our culture tends to disregard much of these in favor of a reasoned plan of procedure based upon understanding, generalization, experimentation and generalization of results. The methods of thinking developed by modern science have brought out many weaknesses in our social and political life. Therefore many people at heart do "not like science." They do not understand the methods of science and so are unwilling to recognize the fact that it dominates our environment.

It is perhaps generally recognized that one cause of our difficulties is our failure to control the forces released by science. If this is to be done effectively, people as a whole must have a conception of science and the methods of science. This is not obtained by reading about it, but by actually doing some scientific work. Someone has said that when civilization has become so complex that the man in the street does not know what it is all about, then the seeds of destruction of that civilization have been sown. If our civilization is to continue and grow, science is valuable to all people trained in our secondary schools, in fact, from a social point of view it is essential.

In a group of this kind I do not need to tell you that mathematics is the queen of the sciences and an elegant vehicle for this scientific training. It is quite possible that some of the opposition to mathematics in our secondary schools has come from those who are afraid of science and the methods of science because they do not understand them. They find proof for their criticism in the work of those of us whose training is so poorly balanced that we lack enthusiasm for and power in our subject. As a result they are attempting to glorify those phases of culture which were dominant in the pre-industrial era. They are inclined to take as an ear mark

of culture a knowledge of the classics. Or, what is more dangerous, they are content to accept for educational purposes, superficial methods and superficial training.

I believe that many of our students are ready for the excellent training they could receive from well prepared teachers, for many of them are constantly on the alert for information. Some of them appear bored, indifferent and superficial. Sometimes the very best are in the greatest mischief. A teacher with power in and enthusiasm for his own subject matter is the only kind who can improve this situation.

In conclusion, because of the civilization and culture of our times, there is need for excellent teaching in mathematics. To meet this need properly, it is essential that our teachers are well trained in general subject matter, in mathematics, in the allied fields and in professional work. Training in one or two of these and a lack in the others is not sufficient. Our teachers must be well prepared to meet the educational needs of today and tomorrow. Not all of this can be achieved by mass movement, nor can it be fully attained by passing laws for minimum requirements for all teachers of our subject. In the final analysis, the responsibility falls on each one of us individually. Each one of us must take stock of himself, discover his weaknesses and then do everything within his own power to improve the situation.

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The Human Side of Mathematics*

By MARY A. POTTER
Racine, Wisconsin

IF WE scrutinize mathematics carefully looking for its human side, we shall observe that in its teaching we find the human element displayed from several different angles. Teachers are, or at least they should be, human beings; pupils are younger human beings; mathematics is a priceless heritage of knowledge developed by the human mind to meet the requirements of human needs. Hence, the teaching of mathematics is the imparting of this heritage of human knowledge to human beings by human beings.

"Yes," I hear one of your pupils remark. "That's all right for you to say because you teach mathematics. But I'm not going to college, so why should I take mathematics?"

Another chimes in, "I am going to college, but I'm not going to be an engineer so why should I study mathematics?"

And yet another, "Mathematics is too hard for me; I never did understand it anyway. Catch me taking any more than I have to."

Then another may add his voice: "My father says I should take something practical like commercial work. He studied algebra and geometry and they never did him any good."

Painful as it is to admit the fact, we shall have to grant that even in the adult world our respected friend, mathematics, is not always considered human. Who doesn't know the teacher of other subjects who prides herself upon not knowing mathematics? And who among you has not been compelled to demonstrate repeatedly that you are not only a normal, but, at times, even an agreeable human being in spite of the fact that you teach mathematics?

That mathematics has fallen into such ill-repute is partly the fault of its friends who, secure in the knowledge of the important role it plays in business, in manufacture, in science, in invention and mindful of its long history as a study of prime importance in the education of the youth of all nations in all ages, have failed to recognize the fact that America believes in advertising and trusts advertised products. The public knows that this is a mechanical age and lauds our great achievements in mechanics; the mathematics teacher knows that mechanics is but applied mathematics and forgets to mention the debt that invention owes to mathematics. The public knows that mathematics is hard and abstract; and since bad news is printable news, the public wails in print against the uselessness and difficulty of the subject taught in our schools; the mathematics teacher is so busy attempting to clarify the abstract principles that he fails to point out their practical application and power. Also, many facts, attitudes, habits and modes of thought—distinctly mathematical—become so ingrained in the minds of the public that they do not regard them as mathematical since they are not bristling with x's and y's. An amusing little example of this fact was recently given by one of our boys in a program he was presenting to our local group of Einsteinites. In explaining how an aviator steered his course and gauged his speed, he explained that the aviator measured the direction and force of the wind, decided upon the direction and velocity he wished to travel, and then by substituting these numbers in a rather complicated formula, he could compute the direction and speed that the plane would need to take. "But," added our informant with a twinkle in his eye as he looked at me, "as a matter of fact, in practical work he does

* Read before the Denver Meeting of the National Council of Teachers of Mathematics on July 1, 1935.

not use mathematics; he does it like this." Thereupon he drew on the blackboard an excellent scale drawing picturing the parallelogram of forces which he solved graphically. Our young friend had made scale drawings since he was in the seventh grade, and now in the twelfth he had forgotten that method of solution as mathematical and recognized it only as a labor saving device.

In these days when everybody's children are being educated, often parents did not attend school long enough to form a very accurate idea of what is contained in the subjects their children are studying. These misconceptions are strange enough that they form the foundations for many stories like the following which may have been true. A mountaineer led his tall son into the country high school and presented him to a teacher he met in the hall.

"This here boy's a comin to school to git larnin," he announced. "What kin you teach him?"

"In my department," the somewhat bookish teacher responded, "we teach arithmetic, algebra, geometry, and trigonometry."

"That'll do," the father interrupted. "Load him with triggernometry. He's the only poor shot in the family."

Psychologists tell us that it is very rare that a person is created who lacks a specific human capacity—that the abilities of a bright person vary somewhat around a high level, that the various aspects of intelligence of a slow student vary about a low possible level of achievement. Hence, our confreres who boast that they are excellent in every subject except mathematics are usually boasting that they do not have a high degree of scholastic aptitude, or else that as students they did not have a compelling desire to work. And since we as pedagogues are not fond of making either of these boasts, no doubt this form of criticism will cease when the teaching profession as a whole is more highly trained in psychology.

In a recent issue of the *Saturday Evening Post* appeared an acknowledgment of a writer of fiction that testing mathematical ability is testing mental calibre. The heroine was attempting to show to the hero the inferior quality of the brain of the beautiful but dumb morsel over whom he had lost his head. Her only comment was, "Did you ever try her out on simple fractions?"

But there is no doubt that we as teachers are somewhat faulty in meeting the criticism that mathematics is difficult. Once upon a time only the mentally elect attended secondary schools and our traditional courses of study were built for them. Not all of us have modified our outlines to care for the slow of learning as Denver and some of the other leading school systems have done. Mathematics, like gasoline, is high powered stuff; overdoses of gasoline in a Model T Ford do not produce the performance of a V 8, but flood the engine, perhaps stopping it altogether. A suitable amount of gasoline in a Model T engine will generate some power, a satisfactory response for the engine used. But gasoline is powerful stuff and poisonous to those weaker machines which are capable of generating only one horse power and are properly fed with oats. Unfortunately we have often insisted upon a degree of performance from our pupils possible only to the V 8's to the utter discouragement of the Model T's and the mathematical annihilation of the humbler horses. It would be far wiser to modify our course of study to fit the digestive capacity of the minds of our students feeding them much mathematical gasoline, little mathematical gasoline, or mathematical oats.

In the well known Report of the National Committee on Mathematical Requirements, the values of mathematics are listed under three divisions—cultural, disciplinary, and practical—all highly desirable human values.

We as teachers have never stressed the cultural aspect of mathematics. We have not classed its pursuit as a leisure time

activity but have rather maintained that, in general, if a person mastered mathematics he would not be in danger of having an overabundant amount of leisure time. But the number of mathematical puzzles being syndicated by the newspapers, and the books of mathematical recreations making their appearance testify to a demand for such pastimes even though their appeal is limited to the mentally competent. In a recent Hobby Show held by the Rotary Club of our town the public library displayed a poster listing favorite hobbies of elementary school children. Taking its place with such old favorites as camping, hiking and stamp collecting, arithmetic was listed. The children's librarian offered the explanation that many boys had told her that they thought it was fun to get together and work out puzzles in arithmetic. I though that their teachers were to be highly commended for making a game out of what is sometimes called drudgery.

Because of their peculiar accuracy, mathematical expressions occupy a unique place in the English language. Without a knowledge of the meaning of mathematical processes, a reader cannot adequately understand such phrases as "difference of opinion," "unequalled beauty," "undivided attention." We take advantage of the power of arithmetical language when we say, "I discounted his statement," "I feel below par today." Since geometry deals with form, we are not surprised to find English strewn with geometric descriptions such as "a square deal," "an obtuse statement," "the eternal triangle," "an unparalleled opportunity," or "an angular build." Even algebra, the chief target for the censure of our critics, offers her contribution in the illuminating of abstract ideas. "Out of proportion," "personal equation," "positive and negative forces" are commonplace. Sum and product—usual as they are in ordinary speech—have implication and distinctions in nicety of thought seldom fully understood until algebra is mastered.

And who can read with comprehension either the text or advertisements of a daily paper or the more staid magazines without a knowledge of graphs? To interpret their full significance, the reader must understand the ideas of ratio and proportion, the implications of relationship, the representation of abstract numbers by concrete lines, bars, circles, or pictograms. From these pictures he is usually supposed to make his own generalizations. All of which, you will admit requires a high degree of mastery of many principles of algebra. Perhaps you can remember when graphing was considered too difficult a subject for secondary school students to study. A few years ago in a safety first campaign a local judge wanted a graph to show to violators of traffic laws in his court the number of feet it required to stop an automobile travelling at different rates of speed. He said that he was accustomed to send to the University (of which he was a regent) when he wanted such graphs drawn, but wondered if by chance I could make one for him. He was impressed when I assured him I could construct it; but he wouldn't believe that our students had that ability until I presented him with a graph which was a sample of their work.

Another claim to culture that mathematics may justly make is that she teaches an appreciation of form and design. Nature, the master artist, presents for patterns an arc of a rainbow, a blazing circle of a sun, trees perpendicular to the ground, parallel blades of grass, spheres of oranges, lacy hexagons of snowflakes, cones of carrots—myriads of geometric forms that children love to discover in the things they know.

If you have ever examined designs made by primitive people, you have been impressed by their infatuation with the straight line. The triangle, the rectangle, the rhombus, the square, the trapezoid and even angles in countless combinations decorate their pottery, the cloth they weave, their houses and their temples. De-

scriptions of modernistic furniture and buildings sound like pages torn from a geometry book. Pupils are delighted when they are shown the relation between Gothic architecture and the isosceles triangle.

In her book called *A Source Book of Problems for Geometry* published by Allyn and Bacon, Miss Mable Sykes has gathered together an amazing collection of beautiful designs used for tiles, floors, linoleums, mosaics, wallpaper and even the policeman's star, and has shown their exact mathematical construction.

So important is mathematics in the arts that sometimes pupils make absurd claims as was told by an English teacher the other day. After a thorough discussion of the poetry of Shakespeare, she gave a test in which she asked, "What is the meter?" On two papers she received this reply: "The meter is 39.37 inches."

During the past quarter of a century many critics have attacked the so-called disciplinary values to be derived from studying different subjects among which they name mathematics. Several authors have wisely deleted the words "mental discipline" from their lists of benefits to be gained from its study and have submitted some phrases similar to the definition of discipline: "instruction and exercises designed to train to proper mental conduct or actions." Shall we examine some of the phases of proper mental conduct or action in which mathematics is peculiarly fitted to train its followers?

From its nature mathematics offers a compelling invitation to the young human to think in abstract terms about a problem presented to him, to analyze a situation, and to form logical deductions. It affords him a unique opportunity for training in correct methods of generalization. By pursuing its study he may acquire a great respect for truth, for accuracy, for checking the accuracy of results, for independence of thought. He may understand the ideas of relationship, of dependence. It is true that these values may be acquired in greater or less degree from the study of other subjects, but mathematics is exceptionally rich in the chance for such training that it affords. However, if the teacher tolerates inaccurate, poorly prepared work, careless thinking, mere memorization, and neglects to help the student discover the application and power of mathematical modes of thought, we shall all agree that such a study contributes but little to the training of the youth in forming correct mental habits.

Too often superficial understanding gives rise to reasoning as faulty as this incident reveals. A salesman was attempting to convince a superintendent of the efficiency of his book for keeping school records. He finished his argument: "This book will do half your work."

To which the superintendent replied delighted, "Good! Then I'll take two!"

But in its history and development mathematics has been primarily a practical subject. As mankind accumulated possessions he needed numbers with which to count them; as he became a trader, he enlarged his study of arithmetic; when he acquired land as a private possession, he evolved a system of geometry with which to measure it.

Measurement in ancient and mediaeval times was fascinatingly displayed in a series of paintings at A Century of Progress. Among others they pictured the ancient Egyptian at his drawing board measuring in forearm lengths, or cubits, distances on his plans for the pyramids which were being built in the background. The legal rod was being measured in Merrie England as the total length of the left foot of sixteen good men as they were leaving church on a Sunday morning. No more clearly may we see the contrast in exactions in ancient and modern times than by contrasting these crude measures with the refinement of the Johanssen Gauges used to check precision instruments in the manufacture of automobiles which measure accurately to the incredible distance of a millionth of an inch.

The history of measurement is a chapter in the history of industry whose findings have been later incorporated into school curricula.

Due perhaps to the greater spread of education, mathematical machines are now being more generally used. The sextant and transit have long been man's friend; during our life time the adding machine has become common although it is not a recent invention. The slide rule (first called the sliding rule instead of the more familiar slip-stick) was invented in 1683 and still is not as widely used as its efficiency warrants. Micrometer gauges are so commonly used by workmen in industrial centers that they rejoice in the familiar nickname of "Mikes." Recently inventors have presented us with other calculators, with machines for computing correlations, with machines that do complicated mechanical graphing, with various machines that perform specialized types of calculation with many times human speed and with far more than human accuracy. Perchance we could make our instruction in mathematics of greater practical value to our students if we could acquaint them with the advantages of some of the simpler of these mathematical instruments.

One way to a man's heart is through his stomach, but nearly as direct a route is through his pocket book. Nobody any longer questions the fact that an engineer's pocket book is flat unless he has had extensive training in the use of mathematics as a tool subject. No one will question the economic necessity of a knowledge of the four fundamentals of arithmetic. In accord with the times, arithmetic has ceased to be merely academic and has become social minded. Since problems of filling the cistern and other foolish puzzles so cleverly satirized in Stephen Leacock's story of the activities of A, B, and C have been supplanted by practical problems such as those dealing with insurance, taxation, banking, investments, mortgages, and making budgets, arithmetic has entrenched herself as one of our most useful servants.

If you have never examined carefully a handbook of applied mathematics, written for the use of the humbler artisan you would be surprised to observe how complicated and extensive are the mathematical tools they find necessary. Of course workmen cannot have a merely passing acquaintance with the facts they use, they must have enough understanding to master the principles. In one such handbook the usual introductory chapters of theory of arithmetic, algebra, geometry, trigonometry, and mechanics are followed by further instruction in excavations and foundations, concrete, bricklaying, carpentry and building, lathing and plastering, painting, plumbing, heating, machine shop, automobile shop, sheet metal work, electricity, print shop, and business mathematics. Selecting at random brickwork, which does not sound highly mathematical, we listed some of the facts that a man must know to be skilled in brickwork.

1. How to read complicated scale drawings.
2. How to read statistical tables.
3. How to compute accurately with whole numbers, fractions including $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, 1/10, 1/12 and decimals up to three places.
4. How to use ratio and proportion.
5. How to find the area of rectangles and triangles.
6. How to use denominate numbers including measures of length, area, and volume.
7. How to substitute in a formula containing a radical in the denominator.
8. To know that the longest side of a right triangle is called its hypotenuse; that an angle is measured in degrees; that a right angle contains 90 degrees; that in a 45 degree right triangle the sides bear a relationship to the hypotenuse of $1/\sqrt{2}$.

That powerful labor saving device, the formula, is creeping into manufacturing, the trades, the professions, business. When facts become well enough established to

become laws they are quickly transformed into the usable instrument called the formula. Solving a fairly simple formula, however, requires more than a little knowledge of algebra—the idea of representing numbers by letters, substitution, order of fundamental operations, use and meaning of exponents, as well as solving equations.

In his excellent book called *The Administration of Mathematics in Secondary Schools*, Dr. E. R. Breslich devoted a section to Methods of Selecting Materials for Teaching Purposes. In this chapter he summarizes the many studies made on the value to human beings of various phases of mathematics; he lists elaborately topics in arithmetic, algebra, and geometry whose knowledge is essential for other school subjects, for business, for trades, for every day living. This summary is of very great value to any one who is responsible for making a course of study.

But with all our forethought, nobody knows for what industries, what occupations, what social conditions, we are training our youth. The radio, the automobile, the moving picture industries (little more than experiments thirty years ago) have joined the ranks of big business. With the advance of civilization, sailing ships have been supplanted by electrically driven floating palaces; in war men no longer throw stones but fire engines of destruction toward targets they cannot see but whose location is figured by complicated formulas. Perhaps in 1965 the well trained youths of 1935 will be piloting rockets going on week end trips to the moon or setting up improved radio sets communicating with the world of spirits.

During the ages mathematics has constantly demonstrated its human qualities by being very useful to mankind. In modern times it has played an important role in creating soft living; a wide spread mastery of the rigors of its logic would save us from the ill effects of much soft thinking. As a tool subject and as a creator of desirable habits, ideals, concepts and modes of thought it helps all of us to learn to live in the present day civilization and it helps some of us to earn a living. So essential is it to the development of our youth that we as teachers must see that the mathematics of the people, by the people and for the people does not perish in our schools.

WE ARE EMBARKED on a course of action in this country that is not only dishonest but inevitably calamitous in outcome. By our want of honesty, our fear of criticism, especially that of being called undemocratic, we are refusing to acknowledge and then act on that knowledge of the very great differences in learning power of children of the same age. So when children reach the age of fourteen, we give them dishonest, lying credentials and send them to schools labeled highschools to take work for which they are unfitted and for which they never will be, never can be and never should be fitted.

For the past twenty-five years we have been devoting our best energies in the public highschools in salvaging the boys and girls at the lower end of the curve of capacity. Our efforts have been largely fruitless. It is time for us all to discover and put in the way of being educated the best brains of the nation, no matter in what circumstances they may be found. DR. JOHN L. TILDSLEY, Assistant Superintendent Public Schools, New York, N. Y., in the Inglis lecture at Harvard.

Report on the Training of Teachers of Mathematics

By Professor E. J. Moulton
Northwestern University, Evanston, Illinois

Teachers of mathematics will be interested in a report on "The Training of Teachers of Mathematics" published in a recent issue of *The American Mathematical Monthly*.[1] The report was prepared by a group of ten mathematicians serving as a Commission on the Training and Utilization of Advanced Students of Mathematics.[2] In December, 1934, it was presented to the Trustees of the Mathematical Association of America by whom it was accepted and approved. The following excerpts from the report may serve to indicate its character.

Preliminary Statement

In a preliminary statement the report states:

As a background for some of the work of this Commission it was recognized that with rare exceptions advanced students of mathematics have always looked to the teaching profession as their means of livelihood. Hence, it is appropriate from time to time to test the orientation of advanced instruction in mathematics when this training is considered as a means for preparing students to be teachers as apart from their possible service in research. In harmony with this point of view, the Commission decided to formulate suggestions for graduate training for teaching mathematics and to make recommendations with regard to the undergraduate training of those who plan to teach mathematics in the secondary schools.

The first part of the report pertains to graduate training for teachers of mathematics. The Commission's point of view is indicated in the following introductory paragraphs:

In the discussion of graduate training, we shall be thinking primarily of preparation for teaching mathematics in a college rather than at a more elementary level. However, the training which we shall outline will be so rounded that it should likewise be recognized as furnishing a student with superior preparation for teaching secondary mathematics, provided that he also adds to his equipment whatever auxiliary knowledge is peculiarly necessary for elementary teaching. We shall assume that the graduate training for teachers should consist of three years of advanced study under the guidance of a department of mathematics, and not under the control of some outside agency such as a college of education. Any first-year graduate curriculum, perhaps leading to a master's degree, will be thought of principally as the first part of a complete program.

A three-year period is specified for the full training because, even for teaching at the secondary level, we believe that a knowledge of mathematics and related fields beyond what is currently represented by a master's degree, particularly if this knowledge is broad rather than specialized, is of assistance to the teacher of mathematics both as a teacher and as a member of his intellectual community. We agree that the level of intelligence demanded for successful completion of this three-year training should be at least as high as that required for the existing doctorate in mathematics. We specify this level regardless of any final decisions which may be reached concerning alterations in the training for the degree of Doctor of Philosophy or concerning the inauguration of a graduate curriculum leading to a new degree.

[1] *The American Mathematical Monthly*, May, 1935. Vol. XLII, No. 5, pp. 263–277. Reprints of this report may be obtained by addressing Secretary W. D. Cairns, Oberlin, Ohio.

[2] E. J. Moulton, chairman, Northwestern University; William Betz, University of Rochester; W. L. Hart, University of Minnesota; J. O. Hassler, University of Oklahoma; E. R. Hedrick, University of California at Los Angeles; E. V. Huntington, Harvard University; M. H. Ingraham, University of Wisconsin; R. G. D. Richardson, Brown University; H. E. Slaught, University of Chicago; E. B. Stouffer, University of Kansas. Five of these men have served as presidents of national mathematical organizations, four as chairmen of mathematics departments in large state universities and three as deans of graduate schools. The report was prepared by a sub-committee of which Professor Hart was chairman, and was approved by the Commission as a whole.

At such a level, the present facilities for graduate instruction and the available student material could be expected to produce annually about one hundred teachers of mathematics with a degree meriting at least as great respect as the present doctor's degree. To judge whether or not this rate of production would be appropriate, we note that there are more than three thousand persons engaged in teaching mathematics above the secondary level in the United States and Canada. Hence, after the present evil effects of reductions in college teaching staffs disappear, the normal replacement demand for teachers of mathematics in the best secondary schools and at higher levels should be sufficient to absorb the teachers whom our three-year curriculum would aim to produce.

In the preceding statements, the word *college* should be understood to refer to any institution of learning offering instruction in mathematics above the secondary level. Thus, we shall not consider preparation for teaching in a junior college as a problem distinct from that of a college or university which offers the full four years of undergraduate work. This attitude is taken because we believe that the teacher of mathematics in a junior college should have essentially the same background as a teacher of senior college courses. We recognize, however, that a candidate for teaching mathematics in certain junior colleges in order to satisfy existing legal requirements may have to make provision for a few items of training in addition to what we shall recommend.

The Doctor's Degree

The Commission proceeds to discuss characteristics of present doctoral training and the desirability of a second type of three year training. In this connection the Commission presents synopses of two types of training:

In amplification of the preceding recommendation, the Commission offers the following synopses of two varieties of three-year training as a basis for further discussion. The first type, referred to hereafter as Type I, involves the preparation of a research thesis of the current variety. The second, referred to as Type II, might omit the customary kind of research thesis but includes various substitute features. Both types of training involve specialized preparation for teaching mathematics. Gradations between these two types might be used for adaptations to individual students.

Recommended Undergraduate Preparation for Candidacy

An undergraduate major or field of concentration in mathematics; introductory courses in physics, astronomy, psychology, economics, one other science, and one other social science; sufficient additional preparation in one field related to mathematics to meet the prerequisites for elementary graduate work in that field; training in English composition beyond the minimum required for a bachelor's degree; sufficient study of German and French to serve as a basis for learning to read mathematics in these languages.

Graduate Training Common to Types I and II

(1) Foundation courses in each major subdivision of advanced mathematics.

(2) Elementary graduate work in at least one outside field related to mathematics.

(3) Specialized preparation for teaching mathematics:

a. The equivalent of one year of observation and assisting three times a week in various college courses in mathematics which are taught by experienced members of the department of mathematics.

b. Practice teaching in college mathematics under the observation of, and with later criticism by, members of the department of mathematics. This teaching might advisably be done in different courses and should amount to the equivalent of at least a two semester-hour course. The practice teaching should involve participation in the construction and grading of examinations.

c. Guided reading in books and periodicals relating to the theory of teaching, testing methods, and educational research. This reading could be directed either by a member of the department of mathematics or, perhaps, by a person outside the department who appreciates the viewpoint of teachers of mathematics.

(4) A final examination, perhaps both oral and written.

Training Peculiar to Type I

(1) Intensive specialization beyond the foundation material in some large field of mathematics as the basis for later research.

(2) A research thesis of a caliber suitable for publication, where the topic and a minimum of general assistance may be given by a thesis director, but where the responsibility for the development of the topic rests strictly with the candidate.

Training Peculiar to Type II

(1) Additional course work in mathematics and in allied fields, with emphasis on breadth of training.

(2) One or more expository papers of the variety known as *minor theses* in the existing requirements for the doctorate in mathematics in some universities. A thesis of this variety would require the candidate to give evidence of his ability to learn independently and to present in good written form, in a brief period of time, some specified known mathematical results with which he was previously unfamiliar.

(3) A major thesis which would exhibit the candidate's mastery of some field of mathematics and expository ability of high order, although perhaps not research ability. The preparation of this thesis should be the independent responsibility of the candidate. This thesis might be of a historical nature, or it might involve material which, though original from the candidate's viewpoint, might not be acceptable for publication in a leading periodical.

With regard to the explicit training for teaching which is recommended in the training common to Types I and II and which is somewhat novel as judged by current methods of training teachers of college mathematics, the Commission presents the following discussion:

In many universities it is customary to employ graduate students as instructors in elementary courses in mathematics. Such independent teaching by relatively inexperienced persons, even when they are under the outside direction of regular members of the staff, is not a suitable substitute for the training specified in (3a) and (3b), and frequently has ill effects on the mathematical futures of the elementary students who are being taught. We believe that it would be highly desirable for graduate students, however brilliant mathematically, to complete successfully the training described in (3a) and (3b) before they are given independent control of classes in college mathematics.

The guided reading described in (3c) is not intended to justify a requirement of course work in Education. In fact, we are convinced that the objective of (3c) would not be attained if the guided reading were replaced by typical general courses in educational theory. The reading described in (3c) should prepare the candidate to evaluate intelligently or criticize constructively conclusions and methods with which he may later be confronted in the teaching profession.

The Commission raises the question concerning names of degrees to be granted for completion of work of Types I and II. It recommends that the degree of Doctor of Philosophy be awarded for work of Type I, and with regard to Type II it states: "While there may be lack of agreement as to the degree to be awarded for completing the new variety of training, we are convinced that this degree should be a Doctor's degree, preferably Doctor of Mathematics or Doctor of Philosophy. The decision as to the degree to be conferred must rest with the faculty of any school that initiates training of the new type."

THE MASTER'S DEGREE

The Commission continues this report by discussing the requirements for a Master's degree.

We shall conclude our remarks with respect to graduate training by making recommendations in regard to requirements for a master's degree in mathematics to be given at the end of one year of graduate study. Under existing conditions in the teaching profession, such a degree by itself is most useful if it implies certification of exceptionally good preparation for teaching secondary mathematics. For it must be anticipated that in the future a mere master's degree will be of only slight use in a search for placement on a college faculty, regardless of whether the college is a two-year or a four-year institution. However, in some cases the candidate for a master's degree will consider it only as the first milestone on the way to a doctor's degree, and will not be primarily interested in the use of the master's degree as an aid in obtaining a teaching position at the secondary level. Thus, in formulating requirements for a

master's degree, we should have in mind the fact that some candidates for the degree will think of it as a goal in itself, while other candidates will consider the work for the degree only as a part of the three-year training for a doctor's degree.

Any candidate for a master's degree in mathematics should aim to obtain the undergraduate preparation which we outlined in the discussion of the three-year program, except that reading ability in either French or German alone would be satisfactory. Beyond this level of preparation the candidate should be given as broad training as is possible within the limits of (at least) twenty-four semester-hours of graduate work, of which approximately six hours might be devoted to a minor field related to mathematics. It might be desirable, although not absolutely essential, for the candidate to conclude his training by writing a major mathematical paper, although this thesis would be, in general, of an expository nature and not worthy of the name research.

The importance of breadth of training rather than specialization during a first year of graduate study for the usual student makes it advisable to recommend that only a minor fraction, say ten per cent, of his time should be devoted to preparation of a thesis, when one is required. In the assignment of subjects for the thesis as well as in the choice of course work, the candidate for a master's degree who intends to continue his graduate study beyond this stage might be differentiated from the candidate who intends to enter immediately into secondary teaching. A thesis subject involving contacts with the secondary field might be assigned to the candidate with an interest in this direction, whereas the other candidate could be assigned a topic which prepares the way for later advanced work in some field of mathematics.

A candidate who desires a master's degree in preparation for secondary teaching should analyze his undergraduate preparation in fields outside of mathematics in the light of the detailed discussion of undergraduate preparation for teaching secondary mathematics which is to be presented in the second part of this report. Deficiencies in the candidate's undergraduate preparation in content fields and in work needed to meet the legal requirements in practice teaching at the secondary level and in the theory of education should be remedied while he proceeds, perhaps at reduced speed, with his graduate work for a master's degree. Even though a student may expect to go forward immediately with a three-year program of graduate study, he would be wise, under existing conditions, to prepare himself for teaching in the secondary field and follow the program which we have just recommended for the candidate who is pointing his preparation definitely toward that field.

Undergraduate Training for Teaching Secondary Mathematics

The second part of the report is devoted to undergraduate training for teaching secondary mathematics. A conspicuous feature of this portion of the report is the advocacy of breadth of training.

A teacher, to be of maximum service to the community in which he lives, should be recognized as an educated man to whom adult members of the community may turn for consultation on intellectual matters. He should be able to participate in community activities, and assume his share of leadership. Certainly he cannot function satisfactorily if he is notably ignorant in what are commonly regarded as fundamentals of general culture. With these facts in mind we advocate a breadth of training for teachers of mathematics which will insure a degree of familiarity with language, literature, fine arts, natural science, and social science, as well as mathematics.

The practical necessity for a breadth of training is emphasized by consideration of fundamental facts in the administration of secondary schools. In a large school it may be possible for a mathematics teacher to limit his teaching to mathematics, but under emergency conditions he may be called upon to give instruction in other fields, and in smaller schools he is almost certain to be asked to do so as a matter of necessity. Thus, it is highly important for the prospective teacher whose major interest is in mathematics to prepare himself for teaching other subjects as well.

Moreover it all too frequently happens that teachers whose major preparation has been in other fields are called on to give part time instruction in mathematics, particularly in the smaller high schools. For this reason it is important, that we should express ourselves in regard

to the appropriate training and mathematical limitations of those secondary teachers who will perform work in mathematics.

In view of the preceding facts we shall discuss a training program in which mathematics is a minor field and another in which mathematics is the major field. As a guiding principle in regard to teaching in the secondary field, we wish to express the opinion that *no teacher should perform a majority of his teaching in a field for which he does not possess major preparation.*

In an effort to raise the general tone of teacher training, our recommendations will depart from what has been current practice. We believe that the higher standard should be obtainable in the near future in view of the prevailing high level of general education and the shrinkage of opportunities for satisfactory employment of college graduates.

Further recommendations which are of great significance to the teaching of mathematics as a minor field are contained in the following paragraphs:

In considering possible suggestions for training in mathematics as a background for minor teaching in this subject, the Commission is impressed with the need for a distinct elevation in the minimum levels now in effect. In view of modern trends in the content of high school courses, a bare acquaintance with algebraic manipulation is no longer sufficient even for a deadly uninspiring presentation of eighth or ninth grade mathematics. In various courses in mathematics at these levels, it is indispensable for the teacher to have at his command a thorough knowledge of trigonometry, college algebra, and the typical methods of analytic geometry. We also find that eighth and ninth grade mathematics involve considerable content whose background is found in the college courses in elementary physics, statistics, economics, and the mathematics of investment. For a truly inspiring presentation of ninth grade mathematics, particularly to students who may plan to prepare for college entrance, the teacher should appreciate the significance of elementary mathematics in the light of important applications of more advanced mathematics. Such equipment requires far more than the mere minimum of training sufficient for a mechanical presentation of subject matter.

Hence, in our recommendations concerning minor preparation for teaching mathematics below the tenth grade, two levels of training are specified: the lower level may be considered as an *irreducible minimum* suitable under temporary emergency conditions, and the upper level as the *desirable training.* In addition to the preparation suitable for teaching ninth grade mathematics, the instructor of tenth grade mathematics, regardless of whether mathematics is his major or a minor field, should have a knowledge of Euclidean geometry beyond that involved in the typical high school course. In fact, we believe that people with only minor preparation for teaching secondary mathematics should not teach tenth grade mathematics except under temporary emergencies, and that these people should never instruct classes in mathematics above the tenth grade.

We make the following recommendations concerning the teaching of mathematics as a *minor* field at the secondary level:

1. A teacher with only minor preparation in mathematics should teach in this field only if he is associated with a teacher having major preparation for teaching mathematics.[3]

2. For teaching mathematics *below the tenth grade,* the minimum mathematical background (suitable only under emergency conditions should include eight semester-hours of college mathematics above two and one-half units of secondary mathematics, completed with grades above the median level. A *desirable* mathematical background should include also: (a) college mathematics through six semester-hours of calculus; (b) introductory college physics and economics; and (c) an introduction to the mathematics of investment.

3. For teaching mathematics *through the tenth grade,* the minimum training should include all items mentioned in the previous paragraph, together with a college course in synthetic Euclidean geometry in advance of high school geometry. A teacher with this limited preparation, however, should teach mathematics in the tenth grade only under emergency conditions, and never above the tenth grade.

4. The prospective teacher with mathematics as a minor would derive profit from

[3] For legislative purposes, cities below a certain size might have to be excused from such a restriction.

a course in methods of teaching secondary mathematics, even if he is able to include little or no associated practice teaching. It would be desirable to substitute such a course for some of the more general educational theory which otherwise might be studied by the candidate in satisfying legal requirements for a secondary teaching certificate.

With regard to the preparation of teachers of mathematics who make mathematics their major field the report includes the following paragraphs:

We have remarked that, in the preparation of a student for major teaching in secondary mathematics, provision should be made for rounded training outside of mathematics in order to provide him with teaching minors. Moreover, breadth of training is essential in giving the future teacher the necessary background for inspirational teaching in his major field. We believe that proper provision for background and cultural courses in the training of a teacher should take precedence over added acquaintance with his major field beyond a suitable level, and over elective courses in the theory of education.

Our recommendations will indicate two levels of preparation in content, one specified as a *minimum* and the other as a *desirable* level. This procedure is adopted because, in certain colleges, requirements in the theory of education and in other fields make it impossible for the desirable level to be attained within the four years of work leading to a bachelor's degree. In such colleges, the student should consider the advisability of completing at least that *desirable* preparation by additional study in a fifth year (which need not necessarily be concluded by the attainment of a master's degree).

In making a recommendation concerning training in educational theory and practice teaching, we shall not approve by name any of the variously labeled courses which appear in the requirements in Education for the high school teaching certificate in different universities. Among such course names we might mention Educational Psychology, History of Education, Adolescent Psychology, Educational Measurements, and so forth. In colleges and departments of education as a whole, and sometimes even within the limits of a single college or department, we find that courses with these names are not standardized as to content. Frequently these courses involve great duplication of material. Moreover, from the standpoint of a major student of mathematics, the statistical content and other theory of a mathematical nature involved in the courses is sometimes presented on such a low level that the student's time is inefficiently used.

The utility of these courses in Education probably depends on the qualifications of the instructors and their ideals more largely than is the case in fields where the subject matter is more definitely standardized. However, we believe that in the fields of pure psychology and educational theory there is much material which should be valuable as training for teachers of secondary mathematics. We are inclined to think that, outside of foundation work in psychology, all of the theory of education presented to the candidate for a secondary teaching certificate in mathematics could best be given in courses definitely oriented with respect to his major teaching field and containing only students whose major or minor interests are in this field.[4]

Recommendations Concerning Preparation for Major Teaching of Secondary Mathematics

(1) *Minimum* training in mathematics.
   a. Courses in mathematics including complete treatments of trigonometry, college algebra, analytic geometry, and six semester-hours of calculus.
   b. A college treatment of synthetic Euclidean geometry (or, possibly, descriptive geometry) (three semester-hours).
   c. Advanced algebra, such as the theory of equations (three semester-hours).
   d. Either directed reading or a formal course in the history of mathematics and the fundamental concepts of mathematics.

(2) *Minimum* college training in fields related to mathematics.
   a. Introductory courses in physics and in another science (twelve semester-hours).
   b. A course in the mathematics of investment (three semester-hours).

[4] This method, though at variance with the common practice of presenting educational theory from a general viewpoint, is in successful operation at the University of Wisconsin in the training of teachers of secondary mathematics.

c. An introductory course in economics.
d. A first course in statistics, with a mathematical viewpoint (three semester-hours).

(3) *Desirable* additional training in mathematics and related fields.
a. Advanced calculus and differential equations or mechanics (six semester-hours).
b. Additional work in geometry, such as projective geometry, solid analytic geometry, etc. (three semester-hours).
c. Additional study in algebra (three semester-hours).
d. Introduction to astronomy.
e. Additional study of physics and other sciences to complete a background in three or more sciences (nine semester-hours).

(4) Adequate training in English composition and cultural training outside of mathematics and related fields. Work in languages, literature, fine arts, and the social sciences in preference to increased specialization in mathematics and related fields, and in preference to elective work in the theory of education beyond the legal requirements.

(5) Training in the theory of education and practice teaching.
a. A one-year course in methods of teaching and practice teaching in secondary mathematics, together with any distinctly pertinent material concerning educational measurements and other content from educational theory (ten semester-hours). It is our belief that this essential part of the student's training should, if possible, be under the direction of professors who have had graduate mathematical training, who have taught mathematics at the secondary level, and who have maintained contacts with the secondary field.
b. Study of methods of teaching in the principal minor field selected by the student and any additional material relating to the history, psychology, or administration of education which can be objectively justified in the training of a teacher (not more than five semester-hours).

Note: In view of the debatable nature of certain features of our recommendation concerning training in the theory of education and practice teaching, the candidate for a teaching certificate in secondary mathematics is advised of the necessity for satisfying the legal requirements in educational theory as they exist in his locality. We believe that further work in the theory of education beyond this legal minimum, by either an undergraduate or a graduate student, would not be as valuable in preparation for teaching mathematics as additional study of mathematics, related fields, and purely cultural subjects. The student is advised to continue study in these fields beyond the undergraduate level along the lines suggested in Part I of this report.

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Interrelationships Between the Purposes, Content and Teaching Techniques of Ninth and Tenth Grade Mathematics

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I. IMPORTANCE *of Stating Clearly a Point of View in the Teaching of Ninth and Tenth Grade Mathematics.*

1. A commitment to *a point of view*, at the outset, is important in any discussion of the teaching of mathematics in the ninth and tenth grades. A background point of view is sketched in the first three or four pages of the present treatise. In the case of actual daily schoolroom practice, the purposes, the content and the technique of teaching ninth and tenth grade mathematics should never be absolutely demarcated. However, very frequently in the past, as well as at the present time, the content offered in mathematics has been, and now is, considered and treated apart from both well-defined desirable purposes, as well as apart from the very best methods of classroom instruction.

2. *The chief thesis of the present discussion is, that for convenience of analysis, theoretically considered, purposes, content and the methods of instruction of ninth and tenth grade mathematics may be separately treated and evaluated, and, then in actual practice all three of these aspects should be considered simultaneously and interrelatedly.* If the above thesis be granted as worthy of consideration, then it means that some of the past purposes used in the teaching of ninth and tenth grade mathematics ought to be revamped, and, that, correspondingly, some of the content offered, and some of the methods of teaching used ought also to be modified.

3. In the writer's opinion, too much time and energy have often been expended, during the last quarter of a century, in trying to see how elementary mathematics content could be deleted or eliminated from the "program of studies" as a whole, rather than in trying to see how some of it could be reorganized so as to be made to be of most value to all pupils—I repeat to all pupils irrespective of the nature of their life work later on. The assertion is herein made that whenever mathematics is so taught that it is well comprehended and appreciated by pupils, it has just as universal value as have the concepts, for example, of English, social studies and elementary science.

4. It seems, therefore, very important to come to some agreement as to what "elemental concepts" in elementary mathematics are to be considered as important for all pupils, in case, at least, of the ninth grade and below. The writer is fully aware of the fact that the National Council of Mathematics, and other groups or committees have previously raised and discussed this issue in some detail. Despite the good work of such groups, elementary mathematics still continues to be taught aimlessly, perfunctorily, and quite too abstractly to immature ninth grade pupils and below. In just so far as elementary mathematics becomes meaningless, and functionless in the pupil's experience, to that extent it cannot be justifiably retained in courses of study.

One justification may be found for this somewhat detailed preliminary discussion. On the whole, in the past, aims for teaching mathematics, and attempts to determine the nature of the subject matter to be taught, and efforts to formulate the best methods of instruction have not been carried along abreast or simultaneously. And so the outcome has often been, that an inordinate amount of emphasis has been placed upon some one of these three above-named phases, out of all proportion to the others. The most obvious cases of such distortion have occurred, first, in an over-emphasis given to mere subject matter taught as an end in itself, whether

such content be gathered from adopted textbooks or elsewhere; and, in the second place, in a slavish over-emphasis to some one or more methods of classroom instruction. On the other hand, purposes, subject matter, and methods of classroom presentation, like the three "graces" should be regarded as complementary to one another.

5. Simple as the foregoing statements may appear to be, they are very consequential when used as a point of view or point of departure in any effort to adapt and re-adapt continually our elementary mathematics taught in the ninth and tenth grades. Briefly *reiterated*, the general *point of view* so far set forth in this discussion is as follows: *The ninth and tenth grade mathematics and below as now found in vogue in most of our public and private schools needs to be reconstructed, because it is ill-adapted to the common needs and capacities of all the pupils involved at these grade levels. Furthermore, a balanced relative emphasis should be placed upon the purposes of the teaching of elementary mathematics in the midst of present day conditions, as well as upon mathematical concepts that are elemental for all pupils, and finally upon methods of presentation that are adapted to growing and comparatively immature minded pupils, rather than upon any one of the above phases to the exclusion of the others.* Under a guiding point of view of the type herein enunciated, reorganization and not elimination of important elements of ninth and tenth grade mathematics would appear to become the chief desideratum. And this task or challenge, as the present writer sees it, is as yet far from satisfactorily solved.

It is easily imaginable that some one may argue, perchance, that the foregoing discussion represents merely the usual platitudinal treatment, and so called pedagogical verbosity. The writer's rebuttal to this attitude would be that, for the most part, our elementary mathematics as now taught in the ninth and tenth grades is chiefly a bookish-like rehearsal of subject matter as an end in itself, and is largely a memoriter-type and meaningless use of sentences and symbols, arithmetical, algebraic, or geometric in nature. On the other hand, it is the writer's contention that pupils of the ninth and tenth grades and below can be made to feel that mathematics does represent one form of significant human experience, just as other forms of realistic experience can be made to be felt by them through the fields of literature, history, science, music and art. In order that pupils may feel that mathematical "concepts" and "skills" do represent real live-wire experiences—some of them concrete and some of them more abstract in nature—the vital interdependence of purpose, subject matter offered, and methods of instruction used must be fully appreciated by administrators and the teaching corps. And this procedure would be quite different from our present-day usual practice.

II. *Agreed-upon Guiding Principles Should Furnish the Basis of the Organization and Presentation of Ninth and Tenth Grade Mathematics.*—Any general point of view which is announced should be accompanied by certain agreed-upon basic principles, whenever any attempt is made to reconstruct the purposes, the nature of subject matter to be offered, and the nature of the methods of classroom instruction in ninth and tenth grade mathematics. A representative list of such principles is here given for consideration by instructors in elementary mathematics:

1. *In the first place certain appropriate aims or purposes in the teaching of elementary mathematics which are applicable to all pupils enrolled in ninth and tenth grade mathematics classes should be consciously in the minds of all instructors.*

2. *In the second place, the quality or kind of subject matter offered in the ninth grade and below should represent chiefly a common or similar body of knowledge, ideas or concepts, and skills for all pupils.* Briefly stated in another way, this means, that "specialization" in mathematics as such should not be the chief concern in case of

the mathematics offered and taught in the ninth grade and below.

3. *In the third place, whenever purposes, content, and methods of teaching are reorganized, both the grades or years above, as well as those below the ninth and tenth, should be consciously taken into account.* This principle, stated in other words, means, that genuine vertical articulation or unification of the different levels of the pupil's learning process rather than mere stratification should obtain, relative to continuity of purposes, relative to gradual and not abrupt transitions from easy to more and more difficult subject matter to be mastered, relative to a reciprocal relationship of methods of instruction used by teachers above and below the ninth and tenth grades.

4. *In the fourth place, the methods of classroom instruction used in ninth and tenth grade mathematics will need to be adapted to the nature of the aims and subject matter agreed upon, as well as to the various needs and capacities of the pupils.* This is to say, that teachers should recognize the necessity of adapting their methods of instruction to widely different as well as to the common needs and interests of pupils in ninth and tenth grade mathematics.

5. *In the fifth place, contrary to most of our past practice, the principle is here announced that all ninth grade mathematics everywhere, whether found either in the four-year high school, or else in the three-year junior high school, should be freed from the slavish dictation of the higher institutions of learning.* This principle involves a modification of our usual, and at one time our well-nigh universal, practice in requiring algebra to be taken, especially in the four-year high school, in the first year or the ninth grade. This principle, be it understood, is not announced in order to disparage in any way the value of mathematical concepts involved in algebra, but is stated in the light of what we ought to set up as some of the outstanding aims in teaching reorganized elementary ninth grade mathematics.

6. A sixth controlling principle here announced perhaps is the most difficult of all to agree upon among expert mathematicians as well as among instructors in mathematics in the ninth and tenth grades and below. In brief, the sixth principle is as follows: *Whenever possible to do so, elemental and fundamental mathematical "concepts" should be taught, irrespective of whether they have been traditionally catalogued or classified under arithmetic algebra, or geometry.* In the writer's opinion much of our past oral and written discussions for and against "unified," "general" or "correlated" mathematics have represented waste effort, because the mere worship of either the "new" or "old" terminologies as such has blinded us to the respective virtues of each. The same difficulty is now again being experienced in the more recent "pro" and "con" discussions of organizing subject matter into the so-called Morrison "teaching units," to which reference will again be made later on in this discussion. Whatever may be one's present attitude of mind toward "unified" mathematics, it is true that part of the crux of the discussion rests on the question as to how far we shall emphasize mathematical "concepts" or ideas fundamental to all study of mathematics in comparison with emphasis either upon mechanical or else meaningful vivisection of mathematics into such subdivisions as arithmetic, algebra, and geometry which was of course originally done for the purpose of convenient labelings and treatment. Apparently, to reiterate a part of the previous discussion, this is a good instance in point, in which content or subject matter has often been considered largely, if not wholly, apart from the aims of teaching, and, from the corresponding technique or method of classroom instruction in ninth and tenth grade mathematics.

Epitomizing, then, six representative basic considerations or principles have been suggested in the foregoing statements. Couched in interrogative form they are as follows:

1. How far is it possible to enlarge the aims of the teaching of ninth and tenth grade mathematics and below so as to be most profitable to all pupils enrolled?

2. How far is it possible to reorganize and modify the traditional nature of the content of ninth and tenth grade mathematics so as to make a larger appeal to all pupils, in terms of their functional and non-specialized knowledge and skills?

3. How far is it possible to unify the mathematical concepts and skills throughout the whole range of their educative process above and below the ninth and tenth grades?

4. How far is it possible for instructors to realize in practice the need for adapting classroom instruction to a wide variety of individual needs and capacities of pupils as well as caring for their common needs and capacities?

5. What should be the relative balance or proportionate emphasis in the ninth and tenth grades between recognizing the common and individual needs of pupils in comparison with the dictates of higher institutions of learning?

6. Is it possible that the discussions relative to so-called "unified," "correlated," or "general" mathematics especially in case of the seventh, eighth, and ninth grades in comparison with the emphasis given to separate subjects such as arithmetic, algebra, geometry, and elementary trigonometry have over-emphasized merely the content as an end in itself rather than to have considered fundamental mathematical concepts and skills in their relation to the aims, and the methods of teaching of mathematics?

III. *Certain Practical Problems Must Be Faced in the Organization and Administration of Ninth and Tenth Grade Mathematics.*—In the light of the general point of view set forth above, and, the light of the six samples of guiding principles announced previously certain practical problems at once arise in one's mind. A number of possible procedures will at once appear to high school administrators, and especially to wide-awake and competent instructors of mathematics in the ninth and tenth grades. One of the most likely initial procedures in these grades is the tendency to select and adopt algebra and geometry texts. And if this were the procedure used it is a pretty safe gamble that the point of view or thesis of the present discussion would be distorted or violated. For it is almost certain that the mere content of subject matter of such a textbook or textbooks would become almost the sole consideration. This suggests of course the whole complicated problem of the analysis, evaluation, and selection or adoption of mathematics textbooks for use in the first two years of the four-year high school.

1. *Importance of Considering the Purposes and Functions of the Teaching of Ninth and Tenth Grade Mathematics.*—The above paragraph is intentionally introduced at this juncture of the discussion in this third large division of this treatise in order to suggest that the first or initial step of any junior or senior high school is to formulate clearly what its outstanding purposes or aims in elementary mathematics teaching are to be; first, in the light of local conditions of the school; second, in the light of the contributions which the study of mathematics ought to make to a pupil's whole life career irrespective of his later calling or occupation; third, in the light of what some pupils will do later on in their pursuit of mathematics in higher institutions of learning; fourth, and finally, in the light of what expert mathematicians feel are important points of emphasis whenever a limited number of pupils themselves desire and strive to become some day mathematicians themselves. I submit that thinking of all of these kinds of aims and purposes, or functions if you prefer, and perhaps of some others, conjointly and simultaneously, is different from our usual traditional procedure or set-up in our ninth and tenth grade practices.

Having accepted as justifiable this kind

of initial step or some other than the usual one-sided traditional procedure, a second group of problems naturally arise. These must at once be faced by administrators, supervisors, and especially by instructors well trained in the field of mathematics. One of these practical problems is, platitudinally stated, *What should constitute the kind of content or subject matter to be taught in these two grades or years?* And our reiterated assumption is that it should be done in the light of the array of aims and functions agreed upon for use in the respective schools in question.

2. *Importance of Determining the Kind of Subject Matter to be Offered in Ninth and Tenth Grade Mathematics.*—No attempt is here made to outline in detail all of the subject matter that should be offered in the ninth and tenth grades. Because of what will be later on said with reference to the organization of elementary mathematics from grades one to twelve, and with reference to "units of teaching," the kind or quality of subject matter here mentioned refers to that of the ninth and tenth grades as well as that of the first to the eighth. May I begin with two assertions. First, the kind as well as the amount of mathematical knowledge and skills should be consciously recognized by instructors. Second, those knowledges, concepts and skills should be given largest attention which will be of some value thoroughout the whole series of years of a pupil's study of mathematics both with reference to everyday life outside of the school, and also with reference to the further pursuit of mathematics within the school.

In part at least, we can all agree that during the first six years of school work, the kind of subject matter that should be emphasized should have to do with "counting"; with "reading numbers"; with "the four fundamental operatives of adding, multiplying, subtracting and dividing"; with "simple measurements and scale drawing"; with relationships of integers to fractions, including decimal fractions; with business considerations involving interest, percentage, accounts, etc. During the seventh, eighth, and ninth grades, the elemental mathematical concepts and skills acquired during the first six grades should be further perfected, and mastered. For instance, more accurate linear measurements and drawing to scale can now be done. More complicated interest and percentage calculations, more detailed study of balancing of accounts, of budgeting, saving accounts and so forth can be done. Formulae for finding the areas of rectangles, triangles and circles, and simple measurement of volumes may be begun. A beginning treatment of the kinds and significance of graphs should be made which should be continued on through to the end of the ninth grade. The preliminary and gradual introduction of somewhat more technical treatment of certain mathematical concepts and skills should be begun in the eighth and ninth grades. A still further refinement of early fundamentals should be continued. Taxation problems, promissory notes, different kinds of insurance such as life, fire, sick and accident should be considered. Business organization bonds, installment buying and so forth may be introduced. A beginning of the more technical aspects relative to the metric system, the use and significance of the equation, including during the ninth grade some simple treatment of fractional equations, simultaneous linear, and quadratic. Some further treatment should be given to the nature and significance of graphical representations. During the seventh, eighth and ninth grades some introduction to geometric concepts and skills can be emphasized, in the way of construction work and so forth. Up to the tenth grade the nature of the work pertaining to college entrance requirements should be incidental if included at all. During the tenth grade for those pupils especially who intend to go on to college preparation for college entrance requirements, as now is our practice, cannot be ignored. However, this being true, many

of the mathematical concepts and skills acquired previous to the ninth grade ought still to continue to function in the tenth grade. It will be apparent from the outline of the kind of subject matter suggested for the first nine grades, that this is intended for all pupils whether they go on to college or not, and furthermore that the chief emphasis is placed on mathematical concepts and skills that are important continuously throughout. Under this plan, for example, the nature and significance of the equation, or of the formula, or of graphical representation would be emphasized continuously, say throughout the seventh, eighth, ninth, and tenth grades, rather than merely under any one subject or another such as arithmetic, algebra, or geometry as such. These it is believed do represent elemental concepts and skills which are important in their simple presentation to all pupils below the tenth grade who are enrolled in mathematics.

3. *The Arrangement of Subject Matter into Convenient Divisions and Subdivisions for Teaching is Unending, One Type of which is the Morrison "Teaching-Unit."*—The transition from a discussion of the kind of subject matter to be offered, to a discussion of the many possible types of arrangements of subject matter into suitable divisions and subdivisions, or into main topics and sub-topics is natural. The Morrison type of "teaching units" have become widely discussed and used throughout especially the Middle West region of the United States. "Units of teaching" according to the Morrison plan are built around "comprehensive" or "significant" themes or concepts, along with the consideration of sub-problems under the major "units of teaching." Such a type of organization or arrangement of subject matter (it is claimed) lends itself rather readily for example to the teaching of literature, or history and social studies, or to biological sciences. Detailed discussions of the same may be found in Morrison's writings, and in numerous magazine articles published by other authors.

The position taken by the writer of this paper is that this "teaching unit" arrangement of subject matter taught in elementary mathematics represents one of a number of possible desirable procedures, which is superior to the usual traditional plan. The emphasis which has been placed upon elemental concepts and skills in the ninth grade mathematics and below in the earlier part of the present discussion is largely in harmony with the "teaching unit" plan of Morrison. Admittedly the difficulty of organizing algebra and geometry as such into "teaching units" is somewhat greater than in case of earlier grade mathematics, because of the logical sequence from one stage to the next higher. Nevertheless if comprehensive and significant major concepts are kept in mind even in algebra and geometry as such there is some gain. Morrison among other examples of his "unitary" teaching organization of subject matter has given the following in algebra: "The concept of literal numbers"; "the concept of plus and minus"; "graphic representation"; "the equation"; "the concept of factors"; "linear" and "quadratic equation." Examples are given also by him for "correlated" mathematics of the seventh and eighth grades. Some of the examples given for plane geometry by him are: "the angle"; "parallel lines"; "perpendicular lines"; "triangles and circles as plane figures."

In case of the subject matter of the ninth grade and below the "unit teaching method" is somewhat easily planned and on the whole is desirable. In case of algebra and geometry as such a modified form of "teaching unit" organization may be found to be desirable by teachers.

4. *Reorganized Mathematics Should Be a Constant in the Ninth Grade and Below, but a Variable in the Tenth Grade.*—Whether the requirement of algebra in its conventional form on the part of all pupils in the ninth grade can be justified is debatable. The case will vary somewhat of course in the light of local conditions. For

example in a case where from eighty-five to ninety or ninety-five per cent of a high school's graduates go on to higher institutions of learning the case is different from that in which less than fifty per cent, or even much lower than this go on to college. Why not recognize these varying conditions? Why not reorganize the nature of the conventional type seventh, eighth and ninth grade elementary mathematics, build it around comprehensive, fundamental concepts profitable to all pupils especially those not going on to college and then require it? Furthermore, require algebra as such of only those pupils planning to go on to college and primarily for the reason that it is still required for college entrance, recognizing at the same time that if it were possible it would be perhaps better to reorganize somewhat, even the algebra and geometry as such now pursued by pupils. In the meantime, is there any good reason why it should not become much more common practice than now is the case to offer algebra in the tenth grade and geometry as such in the eleventh? There is still plenty of time for the pupil to get, in this way three years of mathematics before going to college, and in some instances too pupils will have one more year of mental maturity. The great majority of four-year high schools in Illinois still offer, and many require algebra in the ninth grade. During the year 1933–34, 207 high schools having enrollments of 100 pupils or fewer offered algebra, and prescribed it in the ninth grades; and ninety-seven per cent of the same number of high schools, of the same size, offered plane geometry in the tenth grade; out of eighty-five high schools having pupil enrollments of from 251 to 500, over ninety-five per cent offered algebra in the ninth grade, and over ninety-five per cent offered plane geometry in the tenth grade; out of thirty-six high schools having enrollments from 501 to 1000 one hundred per cent of the schools reported algebra offered in the ninth grade, and one hundred per cent plane geometry in the tenth grade: out of 35 high schools with enrollments of over 1000 pupils from eighty-five to ninety per cent offered algebra and geometry, respectively, in the ninth and tenth grades.[1] The conviction of the writer is that in communities where a small percentage of high school graduates go on to higher institutions of learning the chief purpose or function of elementary mathematics teaching should not point "collegeward," and, that therefore in such localities ninth grade mathematics if of the conventional type of algebra should be made optional, and that in such high schools algebra be made optional in either the ninth or tenth grades for pupils who intend to go to college. Many records of actual practice may be found with reference to the offering, and the non-offering, of algebra in the ninth grade and of geometry in the tenth grade. Stout's study[2] made some years ago showed, for example, for high schools located in North Central Association territory that the inclusion of algebra and geometry was frequent. This study included five-year intervals from 1860 to 1900. In 20 high schools representing 1860 to 1865, 85 per cent offered arithmetic, 90 per cent algebra, and 95 per cent geometry. In the same 20 schools from 1866 to 1870, 60 per cent arithmetic, 100 per cent algebra and geometry, respectively, and this same condition prevailed up to 1880 with the exception of arithmetic. Out of forty schools from 1890 to 1900, about 70 per cent offered arithmetic, 100 per cent algebra, and over 70 per cent geometry.

At the present time some high schools in Illinois require algebra in the ninth grade of all pupils, other schools require algebra only of those pupils who are enrolled in the preparatory college curriculum but make it elective in other cur-

[1] An Investigation of the Program of Studies in Illinois high schools by Clement, Clevenger and post graduate students, based on reports in the High School Visitor's Office.

[2] Adapted from J. E. Stout's—The Development of High School Curricula in the North Central States from 1860 to 1918.

ricula, and some other schools make algebra elective for all pupils enrolled in the ninth grade. Neither space nor time will permit the inclusion of specific high schools which illustrate the above practices.

5. *Leeway Should be Allowed to Instructors in Their Methods of Teaching the Subject Matter of Ninth and Tenth Grade Mathematics in the Realization of Reorganized Purposes and Functions of Different High Schools.*—A variety of profitable methods of classroom instruction should be encouraged among instructors of mathematics in the high schools. Sometimes the methods of instruction used for competent students will need to vary from those used for the less competent. Enslavement to any one single method of classroom instruction is certain to deaden pupils' interests in elementary mathematics especially in the first two years of the high school as well as in the earlier grades. The point of view which has been set forth in this discussion will tend to reduce emphasis on what is known as the mere formal textbook method partly because reorganized purposes and functions and materials of instruction will demand reorganized methods of classroom instruction.

IV. *Practical Implication for Reorganized Teaching of Ninth and Tenth Grade Mathematics.*—What implications for classroom practice grow out of an emphasis on more inclusive purposes and on a more functional content, and on a more vitalized method of instruction in ninth and tenth grade mathematics? In the light of the foregoing discussion the following implications are listed.

1. In the first place, the aims and purposes and functions of elementary mathematics should be more inclusive than mere preparation for doing the work in successive grades or years, including college entrance requirements. This is especially true of the ninth grade and below.

2. In the second place, the kind of content or subject matter that is outlined for ninth grade pupils and below in elementary mathematics should include those concepts and skills that are applicable to all groups of pupils in attendance, both those who will go on with college work and those who will not. During the tenth grade some provision should be made for varying from this practice.

3. In the third place, the whole gamut of the pupil's school experience should be considered when the nature of the mathematics of the ninth and tenth grades is outlined. This continuity of procedure is important both from the learner's point of view as well as that of the instructor's. It is impossible to over-emphasize the importance of this vertical articulation of purposes, content and methods of teaching in elementary mathematics.

4. In the fourth place, whether algebra shall be required in the ninth grade and geometry in the tenth should be determined in part by the conditions of the community in which the school is located. No iron-clad practice should prevail with reference to these two requirements. Some schools, providing it is found to be justified because of local conditions, should have the privilege of either making algebra a constant in the ninth or tenth grade as it chooses, and so likewise in case of offering geometry either in the tenth or eleventh grades.

5. In the fifth place, the exact form of arrangement of subject matter into different divisions or subdivisions, or into so-called "teaching-units" should be left on the whole to individual schools and to competently trained teachers of mathematics. All in all the subject matter of the ninth grade and below will lend itself to the "teaching-unit" organization providing the emphasis is placed on comprehensive concepts of mathematics for all pupils. A modified form of the "teaching-unit" organization can also be used in case of algebra and geometry as such. Whether the more logical sequential divisions which have been used are better or poorer than the use of the "teaching-unit" arrange-

ment will of course depend in part upon the adeptness of the instructor's technique of classroom instruction.

6. In the sixth place, a part of the business of the instructor ought to be to induce the pupil to enjoy, and take an interest in, mathematics, as well as to have him experience the pleasure of mastery over subject matter. The motivating of an abiding interest of some pupils in mathematics may be best done by some instructors by one method of presentation, and by other instructors by other methods of presentation. Furthermore, the adaptation of subject matter when taught to the different capacities of students to learn may justifiably vary among different instructors in mathematics.

7. In the seventh place, administratively considered the newer or composite content to be offered in the seventh, eighth and ninth grades could be offered from three to five times per week. Or if mathematics continued to be offered under the labels of algebra and geometry as such the ninth grade for the pupils not going on to college could be made less intensive than for the pupils who do go on. By the use of an alternating plan either of half years, or years this plan could be effected in the smaller schools as well as in the larger.

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By EDWIN W. SCHREIBER, SECRETARY

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1. Alabama	42	42	36	25	18	21	26	37
2. Arizona	18	10	6	7	7	7	8	10
3. Arkansas	26	25	20	19	20	17	19	20
4. California	130	163	155	130	128	123	130	134
5. Colorado	57	55	50	47	43	52	42	49
6. Connecticut	74	107	89	82	52	62	61	66
7. Delaware	14	8	12	13	10	10	11	9
8. District of Columbia	22	87	86	61	50	47	46	45
9. Florida	18	36	27	31	20	24	24	31
10. Georgia	14	31	33	34	29	35	33	29
11. Idaho	15	10	7	6	7	7	4	6
12. Illinois	287	359	337	287	260	270	277	334
13. Indiana	97	124	94	97	80	86	83	91
14. Iowa	64	152	104	87	71	81	72	74
15. Kansas	102	154	143	106	94	90	102	114
16. Kentucky	28	47	36	39	35	35	27	29
17. Louisiana	19	24	18	17	12	14	14	18
18. Maine	38	30	30	25	23	24	24	21
19. Maryland	86	65	62	62	65	64	52	58
20. Massachusetts	264	262	236	220	196	192	179	194
21. Michigan	104	189	154	133	101	123	120	128
22. Minnesota	143	106	95	102	101	89	80	87
23. Mississippi	21	43	43	34	47	53	53	57
24. Missouri	68	88	87	83	70	72	78	80
25. Montana	8	12	12	13	8	10	14	15
26. Nebraska	48	94	64	78	43	47	50	59
27. Nevada	3	3	7	7	2	2	2	1
28. New Hampshire	17	32	29	25	22	23	22	24
29. New Jersey	138	211	208	205	190	182	167	178
30. New Mexico	5	12	12	12	6	8	10	7
31. New York	292	669	618	567	521	542	515	559
32. North Carolina	55	65	79	73	42	43	45	56
33. North Dakota	15	13	11	9	6	8	7	7
34. Ohio	234	248	215	215	166	174	167	194
35. Oklahoma	61	108	77	66	59	61	64	69
36. Oregon	9	23	18	16	16	18	20	21
37. Pennsylvania	312	513	468	412	358	341	324	345
38. Rhode Island	22	47	38	32	29	29	30	27
39. South Carolina	26	30	20	38	44	46	29	59
40. South Dakota	22	16	18	15	11	10	12	13
41. Tennessee	20	50	38	41	44	48	44	41
42. Texas	75	138	111	112	85	97	93	122
43. Utah	3	11	9	10	12	14	10	12
44. Vermont	18	14	19	15	10	21	22	11
45. Virginia	34	62	60	60	44	46	52	63
46. Washington	28	41	44	31	27	28	27	35
47. West Virginia	31	57	51	44	30	25	39	48
48. Wisconsin	127	112	113	109	96	98	99	102
49. Wyoming	5	17	11	8	9	7	10	10
50. Foreign	53	177	171	139	106	122	135	134
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* The reader will be interested in comparing this table with the frontispiece of this issue.—The editor.